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# Introduction

In the 1720s the European Enlightenment began. This was to be an age of growth in all aspects of life at the time, including state centralization, industrialization, an expansion in overseas empires, population growth, larger military forces, increases in literacy, and expanded learning, notably with a passion for criticism and for major advances in mathematics and the natural sciences.[1] Historians generally date this period as lasting to 1789, the "short eighteenth century." The previous century had brought what has been called the Scientific Revolution, in which mathematics and celestial mechanics were the sciences par excellence. Would they continue to be paramount and their subjects transformed? Where might this occur? How, and why? By late in the century a general assessment came from outside the groups of mathematicians. In the preface to his *Critique of Pure Reason*, published in 1781 with a second edition in 1787, Immanuel Kant expressed the belief that new ideas in mathematics and the natural sciences testified best to the depth of Enlightenment thought.[2] What had happened in the years prior to justify this assessment? The year 1727 marked an important historical moment when, shortly after Isaac Newton died, Leonhard Euler began his distinguished career in Saint Petersburg.

In mathematics at the start of the Enlightenment, many expected few major new achievements or fundamental innovations. The seventeenth century—when most of the field's practitioners came from the aristocracy or from positions in medicine, law, or religion—was considered a golden age of mathematics; at midcentury René Descartes and Pierre de Fermat had each separately created what we now call analytic geometry, and the period culminated in the beginnings of differential calculus in the "method of fluxions" of Newton and the work of Gottfried Wilhelm Leibniz. Many thought that little of general importance was left to pursue.[3] But other scholars anticipated instead a fecund era not only in calculus, including the creation of its core branches, but also across mathematics in theory and application. Above all, the extensive research and writings of Leonhard Euler were to ensure that all of this would occur.

Driven by enormous energy, a passion for mathematics and the exact sciences, a commitment to building a strong institutional base for these



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fields, and an insistent defense of Reform Christianity, Euler diligently pursued an immense research, computational, and writing program across pure and applied mathematics and related technologies from his days in Basel onward except during a few bouts of severe fever. In calculus alone he provided hundreds of discoveries and proofs, along with many fearless computations to simplify and clarify techniques for differential calculus, infinite series, and integral calculus; he was the principal inventor of the core branches of differential equations, together with a semigeometric analytic form and later the analytic calculus of variations. In hundreds of articles and a calculus trilogy starting with the two-volume *Introductio in analysin infinitorum* (Introduction to analysis of the infinite, E101 and E102, 1748),[4] Euler identified foundations; methodically arranged, elaborated, and transmitted calculus; and set out the initial program for calculus's development. As a primary result of his studies, analysis displaced synthetic Euclidean geometry from its two-millennium primacy in mathematics and was the exemplar for reason in the *esprit géométrique* of the period. In pure mathematics Euler did more: he substantially advanced number theory and made headway in algebra, combinatorics, graph theory, probability, topology, and geometry, which included pioneering the differential geometry of surfaces. Drawn deeply also to the exact sciences of mechanics, optics, and astronomy, Euler made contributions across applied mathematics that were unparalleled in combined scope and depth.

Not since the second-century Alexandrian astronomer Claudius Ptolemy had a single geometer so dominated all branches of the exact sciences. Euler was the first to systematically apply calculus to rational mechanics, beginning in 1736 with his two-volume *Mechanica sive motus scientia analytica exposita* (Mechanics of the science of motion set forth analytically, E15 and E16). Prior to the time of William Rowan Hamilton, it was Euler, not Newton, who formulated most of the differential equations in mechanics. Euler founded continuum mechanics and in both print and correspondence led a talented group of competitors and rivals, including Daniel Bernoulli, Alexis Claude Clairaut, Jean-Baptiste le Rond d'Alembert, and Joseph-Louis Lagrange, in transforming into the modern mathematical sciences based on calculus what were then called the mixed mathematics of mechanics, geometric astronomy,[5] optics, dioptrics, acoustics, pneumatics, and games of chance—along with two major fields of physics, physical astronomy and cosmology.[6] Building upon Clairaut's research, Euler proved in theoretical astronomy that Newton's inverse-square law of gravitation by itself accounts for all lunar motion, a major confirmation of Newtonian dynamics. In the mid-eighteenth century Euler was unique in

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creating a mathematical language for the exact sciences that would stand for the next two centuries.

During the Enlightenment, Euler was crucial in helping to build European reputations for the new royal academies of sciences in Berlin and Saint Petersburg. Together with the academy in Paris and the Royal Society of London, they now surpassed the universities in scientific research. At the academies in Saint Petersburg and Berlin, Euler interacted beyond mathematics and the exact sciences in culture, economics, law, politics, religion, and society.[7] The transmission and refinement of his work often relied on correspondence, mostly transmitted through the postal service. Three of these academies were in the capitals of the rising powers of the eighteenth century at a time when France remained dominant. Frederick II of Prussia and Catherine II of imperial Russia assigned Euler royal tasks vital to the growth of commerce, trade, exploration, empire, and the centralizing state; these duties included developing a more exact astronomy, cartography, and geodesy. At the same time the goal was to advance the technologies of artillery, shipbuilding, bridge construction, and instrument building—especially that of clocks, thermometers, microscopes, and telescopes, all of which were essential to making discoveries in the sciences.[8] In Berlin Euler criticized Wolffian philosophy before Kant, argued for improving science education on the university level, and devised plans for state lotteries and pensions. A devout Protestant, he defended traditional religion against the skepticism of the *Encyclopédistes* and freethinkers, but he did not experience a crisis of conscience. In the dispute over the principle of least action, Euler gave to Pierre Maupertuis credit that Euler himself deserved as he questioned the new public sphere that was growing in Europe. In journals and other print, the new public challenged old standards set by state officials, censors, churches, and universities.[9] Euler's influence and the Enlightenment were to extend beyond Europe: he became one of the first foreign members of the American Academy of Arts and Sciences.

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